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LINEAR ALGEBRA AND ITS APPLICATIONS

Linear Algebra and its Applications 370 (2003) 375–378
www.elsevier.com/locate/laa

Two-dimensional reductions of the cone of positive diagonal operators in ℓ^2☆

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Received 20 January 2003; accepted 25 February 2003

Submitted by L. Rodman

Abstract

It is shown that there is no two-dimensional orthoprojector in the Hilbert space ℓ^2 for which the reduction of the cone of positive diagonal operators coincides with the reduction of the cone of all bounded positive operators.

AMS classification: Primary 47B65; Secondary 15A48

Keywords: Hilbert space; Diagonal operator

1. Introduction

Let $\mathfrak{B}$ be the set of all bounded linear operators acting on the Hilbert space ℓ^2 (over $\mathbb{C}$ or $\mathbb{R}$). Every operator $a \in \mathfrak{B}$ is represented by the matrix $(a_{kj})_{k,j\in\mathbb{N}}$, $a_{jk} = \langle ae_k, e_j\rangle$, where (e_n) is the standard basis. Let $\mathfrak{B}^+$ be the cone of all positive operators in $\mathfrak{B}$, let $\mathfrak{D}^+$ be the subcone of diagonal operators in $\mathfrak{B}^+$

$$d \in \mathfrak{D}^+ \Leftrightarrow (d_{kj}) = (\delta_{kj}\gamma_j),$$

where δ_{kj} is the Kronecker delta and $\gamma_j \geqslant 0$.

The article is devoted to proving the following assertion:

Theorem 1.1. *For any orthoprojection q in ℓ^2, $\dim q = 2$,*

$$q\mathfrak{D}^+q \neq q\mathfrak{B}^+q.$$

☆ Supported by the Russian Foundation for Basic Research, grant no. 01-01-00129.
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0024-3795/03/$ - see front matter
doi:10.1016/S0024-3795(03)00480-4